ECG Denoising Using a Dynamical Model and a Marginalized Particle Filter

Chao Lin[1,3], Mónica Bugallo[2], Corinne Mailhes[1,3], Jean-Yves Tourneret[3]

[1]TéSA Laboratory, 14-16, Port Saint-Etienne, 31000 Toulouse, France
[2]Department of Electrical Engineering, Stony Brook University, Stony Brook, 11794-2350 NY, USA
[3]IRIT/ENSEEIHT, University of Toulouse, 2 rue Charles Camichel, 31071 Toulouse, France
chao.lin@tesa.prd.fr, monica@ece.sunysb.edu, {jean-yves.tourneret,corinne.mailhes}@enseeiht.fr

***Abstract*—The development of robust ECG denoising techniques is important for automatic diagnoses of cardiac diseases. Based on a previously suggested nonlinear dynamic model for the generation of realistic synthetic ECG, we introduce a modified ECG dynamical model with 18 state variables to further include morphology variations. A marginalized particle filter is proposed for tracking this modified nonlinear state-space model which has linear substructures. Quantitative evaluations on the MIT-BIH database show that the proposed algorithm outperforms the extended Kalman filter-based algorithms and can better handle non-Gaussian distributions.**

***Index Terms*—Marginalized particle filter, ECG dynamical model, denoising, extended Kalman filter.**

I. INTRODUCTION

The monitoring and analysis of electrocardiograms (ECGs) has received increasing attention because of its vital role in many cardiac disease diagnoses. The development of new sensor technologies has provided new ways of recording ECGs that are more comfortable for patients. However, in most cases, increasing comfort can result in signals with reduced quality. For instance, electrodes that are incorporated in garments generally provide signals with a lower signal-to-noise ratio (SNR) and more artifacts than contact electrodes directly glued to the body [1]. Therefore, extraction of pure ECG components (P, QRS and T waves) from noisy measurements is still a subject of major importance.

A nonlinear dynamical model has been recently developed for the generation of synthetic ECG complexes with their relationship to the beat-to-beat RR-interval timing [2]. Ever since, a particular attention has been devoted to this model whose parameters can be estimated with nonlinear Bayesian filtering. In the literature, one can find several extended Kalman filter (EKF) based ECG denoising techniques [3]–[5]. Earlier work [3] consider the polar form of the dynamical model proposed in [2] and take into account two state variables. In [4], [5], the EKF structure has been modified by considering 15 additional equations to better describe the dynamics of model parameters and improve SNR. However, as pointed out in [6], the EKF always approximates the posterior density at every time instant by a Gaussian density. If the assumption does not hold (e.g., if the true density is bimodal or heavily skewed), sequential Monte Carlo (SMC) methods (often referred to as particle filters (PFs)) can be applied to estimate the joint posterior state distribution. This is precisely the solution investigated in this paper for ECG denoising.

One can find detailed introductions to PFs in [7]. The key idea is to represent the required posterior density by a set of random samples with associated weights and to compute parameter estimates from these samples and weights. As the number of generated samples increases, the resulting Monte Carlo approximation becomes closer to the actual posterior distribution of interest. Despite the simplicity of the PF principle, its main drawback is its computational complexity especially for large state dimension. This computational complexity can be reduced for nonlinear dynamic models containing a subset of parameters which are linear and Gaussian, conditional upon the other parameters. In this case, the linear parameters can be optimally estimated through standard linear Gaussian filtering. This technique is often referred to as Rao-Blackwellization [8] or marginalization[9].

In this paper, we introduce a dynamic model with 18 state variables that allows the artificial ECG to adapt to normal and abnormal morphologies. Since these state equations are linear with respect to a subset of the unknown parameters, we propose to use a marginalized particle filter (MPF) that gets rid off the states appearing linearly in the dynamics, generate particles in the state of the remaining states and run one Kalman filter for each of these particles to estimate the "linear" parameters. The proposed MPF is evaluated on both synthetic signals generated by [2] and real ECG signals from easily available standard databases. A quantitative comparison shows that the proposed MPF outperforms the classical EKF-based denoising techniques in terms of SNR.

The paper is organized as follows. A brief introduction to the modified ECG dynamical model is provided in section II. Section III is dedicated to the description of the proposed MPF algorithm for ECG denoising. Simulation results are provided in Section IV. Discussion and conclusions are finally reported in Section V.

II. ECG MORPHOLOGY AND ECG MODEL

As displayed in Fig. 1, each beat of the heart can be observed as a sequence of deflections away from the baseline of the ECG. A normal ECG cycle consists of five major components contained in the complex PQRST. The first deflection (P-wave) is due to the depolarization of the atria. The large QRS complex is due to the depolarization of the

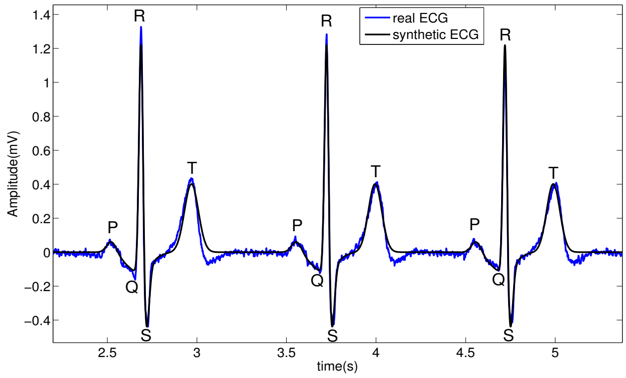

Fig. 1. Signal portion from a normal sinus ECG record (blue) and the synthetic ECG generated by the dynamical model proposed in [2] (black).

ventricles. The last deflection (T-wave) corresponds to the ventricular repolarization of the heart.

The ECG dynamical model proposed by McSharry *et al.* consists of a three dimensional state equation, which generates a trajectory with the Cartesian coordinates [2]. This dynamic model generates a trajectory with the coordinates (x, y, z) according to the following system of differential equations

$$\begin{aligned} x' &= \alpha x - \omega y \\ y' &= \alpha y + \omega x \\ z' &= - \sum_{j \in \{P,Q,R,S,T\}} a_j \Delta\theta_j \exp\left(-\frac{\Delta\theta_j^2}{2b_j^2}\right) - (z - z_0) \end{aligned}$$

where x', y' and z' are the derivatives of x, y and z with respect to time. Here, $\alpha = 1 - \sqrt{x^2 + y^2}$, $\Delta\theta_j = (\theta - \nu_j)\mathrm{mod}(2\pi)$, $\theta = \mathrm{atan2}(y, x)$ and ω is the angular velocity which can be defined as $\omega = 2\pi/T$ with T the RR peak period in each ECG cycle. The baseline wander of the ECG signal has been modeled with z_0. The projection of the three dimensional trajectory on the z axis gives us a synthetic ECG signal. The parameters a_j, b_j and ν_j with $j \in \{P, Q, R, S, T\}$ can be assigned different values to synthesize different ECG signals. The synthetic signal shown in Fig. 1 (black curve) has been generated by the dynamical model whose parameters have been estimated to fit the actual ECG signal (blue line).

In [3], Sameni *et al.* transformed the previous system of differential equations into a simplified discrete polar form

$$\begin{aligned} \theta_{k+1} &= \theta_k + \omega\delta \\ z_{k+1} &= -\sum_{j=1}^{J} \delta \frac{\alpha_j \omega}{b_j^2} \Delta\theta_{j,k} \exp\left(-\frac{\Delta\theta_{j,k}^2}{2b_j^2}\right) + z_k + \eta_k \end{aligned} \tag{1}$$

for $k = 1, ..., K-1$, where K is the number of ECG samples and where $\Delta\theta_{j,k} = (\theta_k - \nu_j) \mod (2\pi)$. Here, $\delta = 1/Fs$ is the sampling time, ω is the angular velocity and η_k is a random time variant noise which has been placed to represent the baseline wander. The summation over j is taken over the number of Gaussian functions (or turning points) J used for modeling each of the ECG component. In [3], the author proposed to use five Gaussian functions to model the ECG channels containing the P, Q, R, S and T waves such that $J = 5$. Moreover, the corresponding model parameters $\boldsymbol{\alpha} = [\alpha_1 \ \ldots \ \alpha_5]^T$, $\boldsymbol{b} = [b_1 \ \ldots \ b_5]^T$, $\boldsymbol{\nu} = [\nu_1 \ \ldots \ \nu_5]^T$ were fixed as presented in Table I.

TABLE I
FIXED ECG FEATURE PARAMETERS PROPOSED IN [3]

Index (j)	P	Q	R	S	T
α_j	1.2	-5.0	30	-7.5	0.75
ν_j	$-\pi/3$	$-\pi/12$	0	$\pi/12$	$\pi/2$
b_j	0.25	0.1	0.1	0.1	0.4

III. MODIFIED ECG DYNAMICAL MODEL

The state variables considered in [3] were θ_k and z_k whereas the other parameters were considered as noise processes. Exploiting the fact that ECG complexes originated from consecutive heartbeats are very similar but not exactly identical, the parameters $\boldsymbol{\alpha}$, $\boldsymbol{b}$ and $\boldsymbol{\nu}$ (defining the five Gaussians used to model the different waves) were considered as state variables with first order autoregressive (AR) dynamics in [4]. In this paper, we propose to consider the angular velocity ω as a new state variable to make the artificial ECG adaptable to rhythm changes. The dynamical state equations for θ_k and z_k can then be written

$$\begin{aligned} \theta_{k+1} &= \theta_k + \omega_k\delta + e_{\theta,k} \\ z_{k+1} &= -\sum_{j=1}^{5} \delta \frac{\alpha_{j,k}\omega_k}{b_{j,k}^2} \Delta\theta_{j,k} \exp\left(-\frac{\Delta\theta_{j,k}^2}{2b_{j,k}^2}\right) + z_k + e_{z,k} \end{aligned} \tag{2}$$

where $e_{\theta,k}$ and $e_{z,k}$ are additive Gaussian noises. In order to consider the time variations of the angular velocity and of the parameters of the different waves, we propose to use the following random walk dynamics for these parameters

$$\begin{aligned} \omega_{k+1} &= \omega_k + e_{\omega,k} \\ \boldsymbol{\alpha}_{k+1} &= \boldsymbol{\alpha}_k + \boldsymbol{e}_{\alpha,k} \\ \boldsymbol{\nu}_{k+1} &= \boldsymbol{\nu}_k + \boldsymbol{e}_{\nu,k} \\ \boldsymbol{b}_{k+1} &= \boldsymbol{b}_k + \boldsymbol{e}_{b,k} \end{aligned} \tag{3}$$

where $e_{\omega,k}, \boldsymbol{e}_{\alpha,k}, \boldsymbol{e}_{\nu,k}$ and $\boldsymbol{e}_{b,k}$ are additive mutually independent white noise vectors whose variances determine how fast the parameters are expected to change with time. The resulting ECG signal is defined by 18 state variables gathered in $\boldsymbol{x}_k$ and by 12 process noise variables defining $\boldsymbol{w}_k$

$$\begin{aligned} \boldsymbol{x}_k &= \left[\theta_k \ z_k \ \omega_k \ \boldsymbol{\alpha}_k^T \ \boldsymbol{\nu}_k^T \ \boldsymbol{b}_k^T\right]^T \\ \boldsymbol{w}_k &= \left[e_{\theta,k} \ e_{z,k} \ e_{\omega,k} \ \boldsymbol{e}_{\alpha,k}^T \ \boldsymbol{e}_{\nu,k}^T \ \boldsymbol{e}_{b,k}^T\right]^T. \end{aligned}$$

Concerning the observation equation, besides the ECG observations z, Sameni *et al.* proposed in [3] to add the phase ϕ as a second observation, which can be simply obtained by detecting the R peaks. Hence, we have two noisy observation vectors $\boldsymbol{\phi} = (\phi_1, \cdots, \phi_K)$ and $\boldsymbol{s} = (s_1, \cdots, s_K)$ (where K is the number of observed samples), corresponding to the state variables θ and z. Other state variables are considered as hidden states. The resulting observation equation of the dynamical model can be defined as

$$\begin{aligned} \phi_k &= \theta_k + u_{\phi,k} \\ s_k &= z_k + u_{s,k} \end{aligned} \tag{4}$$

where $u_{\phi,k}$ and $u_{s,k}$ are observation noises. This work considers white Gaussian noises in order to compare the proposed algorithm with EKF based methods. However, one advantage of the proposed MPF is that it can be modified easily to handle non-Gaussian noises and/or nonlinear state or measurement equations. This will be useful in the presence of motion articfacts, environmental noises and bioelectrical artifacts which can induce non-linearities or/and non-Gaussian noises in the ECG dynamical model.

IV. A MARGINALIZED PARTICLE FILTER

By looking carefully at (2) and (3), it can be observed that the state equation is composed of linear functions of some state variables. More precisely, the state equation has 5 linearly dependent parameters (i.e., $\boldsymbol{\alpha}_k$) and 13 nonlinear parameters (i.e., $\theta_k, z_k, \omega_k, \boldsymbol{\nu}_k^T$ and $\boldsymbol{b}_k^T$). In this case, the "nonlinear" variables can be obtained using a sequential Monte Carlo method whereas the "linear" parameters are estimated using a recursive Kalman filter (KF). This sequential method is referred to as MPF [9]. More precisely, the state vector $\boldsymbol{x}_k$ can be split into two subvectors $\boldsymbol{x}_k^{\mathrm{L}}$ and $\boldsymbol{x}_k^{\mathrm{NL}}$ referred to as linear (L) and nonlinear (NL) components

$$\begin{aligned} \boldsymbol{x}_k^{\mathrm{NL}} &= \left[\theta_k \; z_k \; \omega_k \; \boldsymbol{\nu}_k^T \; \boldsymbol{b}_k^T\right]^T \\ \boldsymbol{x}_k^{\mathrm{L}} &= \boldsymbol{\alpha}_k^T. \end{aligned}$$

Using Bayes' theorem, the following result can be obtained

$$p\left(\boldsymbol{x}_k^{\mathrm{NL}}, \boldsymbol{x}_k^{\mathrm{L}} | \boldsymbol{y}_k\right) = \underbrace{p(\boldsymbol{x}_k^{\mathrm{L}} | \boldsymbol{x}_k^{\mathrm{NL}}, \boldsymbol{y}_k)}_{\text{Optimal KF}} \underbrace{p\left(\boldsymbol{x}_k^{\mathrm{NL}} | \boldsymbol{y}_k\right)}_{\text{PF}} \tag{5}$$

where $p(\boldsymbol{x}_k^{\mathrm{L}} | \boldsymbol{x}_k^{\mathrm{NL}}, \boldsymbol{y}_k)$ is analytically tractable and is given by the KF equations, while $p\left(\boldsymbol{x}_k^{\mathrm{NL}} | \boldsymbol{y}_k\right)$ can be estimated using the PF. In order to highlight the linear and nonlinear relations between the different variables, it is interesting to rewrite the transition and observation equations (2) and (4) in the form of the *Triangular Model* proposed in [9] as follows

$$\begin{aligned} \boldsymbol{x}_{k+1}^{\mathrm{NL}} &= \boldsymbol{x}_k^{\mathrm{NL}} + \boldsymbol{G}\left(\boldsymbol{x}_k^{\mathrm{NL}}\right) \boldsymbol{x}_k^{L} + \boldsymbol{e}_k^{\mathrm{NL}} \\ \boldsymbol{x}_{k+1}^{\mathrm{L}} &= \boldsymbol{x}_k^{\mathrm{L}} + \boldsymbol{e}_k^{\mathrm{L}} \\ \boldsymbol{y}_k &= \boldsymbol{x}_k^{\mathrm{NL}} + \boldsymbol{u}_k \end{aligned} \tag{6}$$

where

$$\begin{aligned} \boldsymbol{G}\left(\boldsymbol{x}_k^{\mathrm{NL}}\right) &= \left[\boldsymbol{0} \; \boldsymbol{g}\left(\boldsymbol{x}_k^{\mathrm{NL}}\right) \; \boldsymbol{0} \; \cdots \; \boldsymbol{0}\right]^T \\ \boldsymbol{g}\left(\boldsymbol{x}_k^{\mathrm{NL}}\right) &= \left[g_{k,1}^{\mathrm{NL}} \; \ldots \; g_{k,5}^{\mathrm{NL}}\right]^T \end{aligned} \tag{7}$$

and

$$g_{k,j}^{\mathrm{NL}} = \delta \frac{\omega_k}{b_{j,k}^2} \Delta\theta_{j,k} \exp\left(-\frac{\Delta\theta_{j,k}^2}{2b_{j,k}^2}\right) \tag{8}$$

The linear part of (7) can be formed for each particle $\left\{\boldsymbol{x}_k^{\mathrm{NL},i}\right\}_{i=1,\ldots M}$, and the linear state variables can be estimated using the KF. This requires to build one KF for each particle. The MPF recursions are summarized in Algorithm 1, and are detailed in the sequel. Note that M is the number of particles and K is the number of time steps which is equivalent to the number of observed samples.

Algorithm 1 The marginalized particle filter

{Initialization}
for $i = 1$ to M **do**
  sample $\boldsymbol{x}_{0|-1}^{\mathrm{NL},i} \sim p(\boldsymbol{x}_0^{\mathrm{NL}})$
  set $\boldsymbol{x}_{0|-1}^{\mathrm{L},i} = \boldsymbol{x}_0^{\mathrm{L}}$ and $P_{0|-1}^i = P_0$
end for
for $k = 0$ to K **do**
  {Evaluate importance weights}
  for $i = 1$ to M **do**
    $w_k^i = p\left(\boldsymbol{y}_k | \boldsymbol{x}_{k|k-1}^{\mathrm{NL},i}\right)$
  end for
  {Normalize the importance weights}
  $\widetilde{w}_k^i = w_k^i / \sum_{j=1}^{M} w_k^j$
  {Resampling}
  Resample M particles with replacement
  $P\left(\boldsymbol{x}_{k|k}^{\mathrm{NL},i} = \boldsymbol{x}_{k|k-1}^{\mathrm{NL},j}\right) = \widetilde{w}_k^j$
  {Particle filter time update and Kalman filter }
  a) Kalman filter measurement update (Linear): see (9)
  b) Particle filter time update (Nonlinear):
  for $i = 1$ to M **do**
    sample $\boldsymbol{x}_{k+1|k}^{\mathrm{NL},i} \sim p\left(\boldsymbol{x}_{k+1|k}^{\mathrm{NL}} | \boldsymbol{x}_k^{\mathrm{NL},i}\right)$
  end for
  c) Kalman filter time update (Linear): see (13)
end for

1) Initialization: The initial values of the model parameters $\boldsymbol{x}_0$ and the process noise covariance matrix P_0 are obtained by using an automatic off-line parameter selection procedure proposed in [3]. By plotting the noisy ECG versus the periodic phases that are assigned to each sample in polar coordinates, the mean and variance of the phase-wrapped ECG can be calculated for all phases between 0 and 2π. This gives the average of the ECG waveform. The best estimates of the model parameters in the MMSE sense can be found by fitting the mean ECG by using a nonlinear least-squares approach and they are assigned to the initial parameters. The process noise covariance matrix can be obtained by calculating the magnitude of the deviation of the parameters of the Gaussian functions in (1) around the estimated mean. Interesting readers are invited to refer to [3] for more details of this parameter selection procedure.

2) Kalman filter measurement update: As shown in (4), the noisy measurements do not depend on the linear state variables $\boldsymbol{\alpha}_k$. Therefore, the measurement update in the KF can be simplified as follows

$$\hat{\boldsymbol{\alpha}}_{k|k}^i = \hat{\boldsymbol{\alpha}}_{k|k-1}^i, \quad P_{k|k}^i = P_{k|k-1}^i \tag{9}$$

where $\hat{\boldsymbol{\alpha}}_{k|k}^i$ and $P_{k|k}^i$ are the mean vector and covariance matrix of the conditional probability density functions (pdfs) for $\boldsymbol{\alpha}_{k|k}^i$.

3) Particle filter time update: The prediction of the nonlinear state variables $\hat{\boldsymbol{x}}^{N}_{k+1|k}$ is obtained by using the PF. In this paper, we adopt the prior distribution as importance distribution, i.e., we sample the particles $\hat{\boldsymbol{x}}^{\mathrm{NL},i}_{k+1|k}$, $i \in 1,\ldots,M$ from $p\left(\boldsymbol{x}^{\mathrm{NL}}_{k+1|k}|\boldsymbol{x}^{\mathrm{NL},i}_{k}\right)$. Using (2), it can be shown that the pdfs $p\left(\theta_{k+1|k}|\theta^i_k\right)$, $p\left(z_{k+1|k}|z^i_k\right)$, $p\left(\omega_{k+1|k}|\omega^i_k\right)$, $p\left(\boldsymbol{\nu}_{k+1|k}|\boldsymbol{\nu}^i_k\right)$ and $p\left(\boldsymbol{b}_{k+1|k}|\boldsymbol{b}^i_k\right)$ are Gaussian pdfs. Meanwhile, we adopt the assumption that the noise sources are uncorrelated as in [3], [4]. Thus, the importance distribution is Gaussian

$$p\left(\boldsymbol{x}^{\mathrm{NL}}_{k+1|k}|\boldsymbol{x}^{\mathrm{NL},i}_{k}\right) = \mathcal{N}\left(\boldsymbol{\mu}_k,\ \boldsymbol{\Sigma}_k\right)$$

with mean vector $\boldsymbol{\mu}_k$ defined by

$$\boldsymbol{\mu}_k = \begin{bmatrix} \theta^i_k + \omega^i_k \delta \\ -\boldsymbol{g}^T\left(\boldsymbol{x}^{\mathrm{NL},i}_k\right)\boldsymbol{\alpha}^i_k + z^i_k \\ \omega^i_k \\ \boldsymbol{\nu}^i_k \\ \boldsymbol{b}^i_k \end{bmatrix}. \tag{10}$$

Note that $\boldsymbol{\alpha}^i_k$ are generated by the KF at instant k. The variance matrix $\boldsymbol{\Sigma}_k$ is given by

$$\boldsymbol{\Sigma}_k = \mathrm{diag}\left(\sigma^2_\theta,\ \sigma^2_{z^*},\ \sigma^2_\omega,\ \boldsymbol{\sigma}^2_\nu,\ \boldsymbol{\sigma}^2_b\right) \tag{11}$$

where $\sigma^2_{z^*} = \boldsymbol{g}^T\left(\boldsymbol{x}^{\mathrm{NL},(i)}_k\right) P^{(i)}_{k|0:k-1}\boldsymbol{g}\left(\boldsymbol{x}^{\mathrm{NL},(i)}_k\right) + \sigma^2_z$, $\boldsymbol{\sigma}^2_\nu = \left[\sigma^2_{\nu_1},\ \ldots,\ \sigma^2_{\nu_5}\right]^T$ and $\boldsymbol{\sigma}^2_b = \left[\sigma^2_{b_1},\ \ldots,\ \sigma^2_{b_5}\right]^T$.

4) Kalman filter time update: It is clear that among the nonlinear state variables, only z_k contains information about the linear state variables $\boldsymbol{\alpha}_k$. This implies that there will be information about the linear state variable $\boldsymbol{\alpha}^i_k$ in the prediction of the nonlinear state variable $\hat{z}^i_{k+1|k}$. It is assumed that the PF time update step in Algorithm 1 has just been completed. This means that the predictions $\hat{\boldsymbol{x}}^{\mathrm{NL},i}_{k+1|k}$ (which include $\hat{z}^i_{k+1|k}$) are available, and the state equations of $\boldsymbol{\alpha}_k$ can be written as

$$\begin{aligned} \boldsymbol{\alpha}^i_{k+1} &= \boldsymbol{\alpha}^i_k + \boldsymbol{e}_{\alpha,k} \\ \hat{z}^i_{k+1|k} - z^i_k &= \boldsymbol{g}^T\left(\hat{\boldsymbol{x}}^{\mathrm{NL},i}_{k+1|k}\right)\boldsymbol{\alpha}^i_k + e_{z,k}. \end{aligned} \tag{12}$$

It is possible to interpret $\hat{z}^i_{k+1|k} - z^i_k$ as a measurement of $\boldsymbol{\alpha}_k$ and $e_{z,k}$ as the corresponding measurement noise. Since (12) is a linear state-space model with Gaussian noise, the optimal state estimate is given by the KF, i.e.,

$$\begin{aligned} F_k &= \boldsymbol{g}^T\left(\hat{\boldsymbol{x}}^{\mathrm{NL},i}_{k+1|k}\right) P^i_{k|k}\boldsymbol{g}\left(\hat{\boldsymbol{x}}^{\mathrm{NL},i}_{k+1|k}\right) + Q^{\mathrm{NL}}_k \\ L_k &= P^i_{k|k}\boldsymbol{g}\left(\hat{\boldsymbol{x}}^{\mathrm{NL},i}_{k+1|k}\right) F^{-1}_k \\ \hat{\boldsymbol{\alpha}}^i_{k+1|k} &= \hat{\boldsymbol{\alpha}}^i_{k|k} + L_k\left(\hat{z}^i_{k+1|k} - z^i_k - \boldsymbol{g}^T\left(\hat{\boldsymbol{x}}^{\mathrm{NL},i}_{k+1|k}\right)\hat{\boldsymbol{\alpha}}^i_{k|k}\right) \\ P^i_{k+1|k} &= P^i_{k|k} + Q^{\mathrm{L}}_k - L_k F_k L^T_k \end{aligned} \tag{13}$$

where $\hat{\boldsymbol{\alpha}}^i_{k+1|k}$ and $P^i_{k+1|k}$ are the filtered estimates of the mean and covariance of the conditional pdfs for $\boldsymbol{\alpha}^{(i)}_{k+1|k}$, respectively. The noise covariance matrices are $Q^{\mathrm{NL}} = \sigma^2_z$ and $Q^{\mathrm{L}} = \Sigma^2_{\boldsymbol{\alpha}}$.

5) Weight update: Since the observation equation does not depend on the linear state variables $\boldsymbol{\alpha}_k$, the measurement noise can be arbitrarily distributed. In this case, (4) is solely used in the PF part of the algorithm, which can handle all pdfs. However, for comparison purposes with the EKF, the results of this paper are based on simple additive Gaussian measurement noises. Therefore, the likelihood $p\left(\boldsymbol{y}_k|\boldsymbol{x}^{\mathrm{NL},i}_{k|k-1}\right)$ is a bivariate Gaussian pdf. The measurement noise covariance matrix is assumed to be diagonal

$$p\left(\phi_k, s_k|\theta_k, z_k\right) = \mathcal{N}\left(\begin{bmatrix}\theta_k \\ z_k\end{bmatrix}, \begin{bmatrix}\sigma^2_\phi & 0 \\ 0 & \sigma^2_s\end{bmatrix}\right).$$

Consequently, the importance weights satisfy the following relation

$$\widetilde{w}^i_k \propto p\left(\phi_k, s_k|\theta^i_{k|k-1}, z^i_{k|k-1}\right)$$

where $\propto$ means "proportional to".

V. SIMULATION RESULTS

In this paper, we use one of the easily available standard databases, namely, the MIT-BIH Normal Sinus Rhythm Database (MIT-BIH) [10] to study the performance of the proposed method. This database was recorded at a sampling rate of 128 Hz from 18 subjects with no significant arrhythmia. From this database, 20 portions of 10 seconds are visually selected for the evaluation of the proposed algorithm. These signal portions have been selected with a high SNR and they constitute our ground truth. In order to generate realistic ECG signals, the ground truth has been corrupted by an additive white noise with SNR ranging from 25 to -5 dB. In order to investigate the performance of our algorithm and to compare it to alternative methods, we have implemented the EKF model of [4] with 17 state variables (which has been shown to outperform other EKF based methods in [4]). In the following simulations, all methods have been tested with the same values of the process and measurement noise parameters, which have been estimated by using the automatic parameter estimation procedure derived in [3].

Fig. 2 shows the results obtained for an ECG portion with P and T wave amplitude variations. We can see that the proposed MPF provides cleaner estimates, especially with smaller distortions on wave peaks. One explanation is that the dynamic ECG model with 18 variables describes more accurately the ECG signal variations. Moreover, the proposed MPF outperforms the EKF for parameter estimation in ECG signals. In order to confirm the visual performance evaluation of the proposed MPF, we have considered the so-called SNR improvement measure defined as

$$\mathrm{SNR}_{\mathrm{imp}} = 10\log\left(\frac{\sum_i |s(i) - x(i)|^2}{\sum_i |z(i) - x(i)|^2}\right)$$

where s is the noisy signal, x is the clean ECG and z is the denoised signal.

In order to obtain a fair performance comparison between the different algorithms, 20 Monte Carlo runs have been considered for each ECG signal portion. The filter output SNR has been averaged over the 400 results for each input

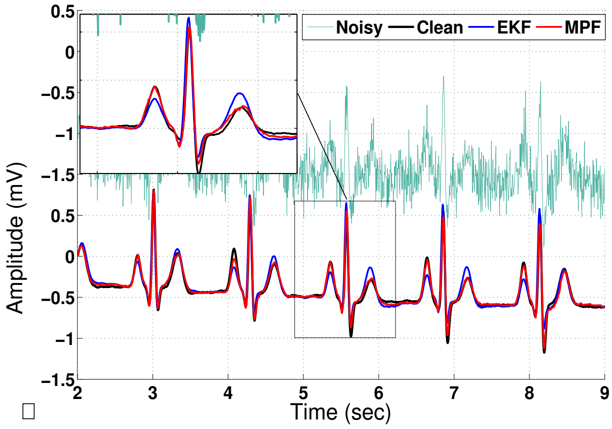

Fig. 2. Typical filtering results of EKF proposed in [4] and MPF with 2000 particles for an noisy ECG signal of 0dB. On the top: input signal; on the bottom: original clean signal (dark green), EKF filtered output (blue) and MPF filtered output (red).

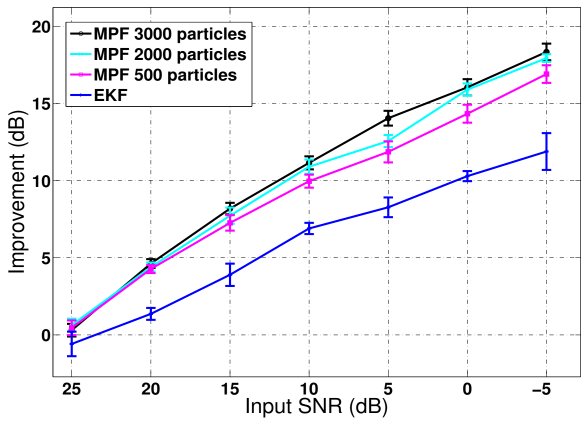

Fig. 3. MPF with 3000, 2000, and 500 particles and EKF filter output SNR improvement versus different input SNRs for 20 portions selected from the MIT-BIH Normal Sinus Rhythm database, averaged over 20 Monte Carlo simulations each.

SNR. The means and standard deviations of SNR improvements versus different input SNRs are plotted in Fig. 3. The proposed MPF algorithm clearly outperforms the EKF method in terms of SNR improvement, especially for low input SNRs. From Fig. 3, it is also clear that the estimates improve when more particles are used as expected. Note that the difference in performance can mainly be observed during the transients, where more particles should be used. Of course, the computational complexity is an increasing function of the number of particles. A good compromise is obtained when using 2000 particles for ECG denoising.

VI. CONCLUSIONS

This paper studied a marginalized particle filter for Bayesian filtering of single channel noisy ECG signals. The marginalization was applied to a modified dynamic ECG model defined by a nonlinear state-space model with linear substructures. A validation using the MIT-BIH database showed that further improvements in estimation performance can be obtained when using a PF instead of the classical extended Kalman filter. Instead of using local linearizations of the nonlinear state-space model (as for the EKF based methods), the proposed MPF approximates the unknown parameter posterior distribution by exploiting the specific structure of the state space model. It it interesting to note that the MPF method allows any (possibly non-Gaussian) noise distribution to be considered. This property is very interesting for physiological signal processing, where the noise is often complex and non Gaussian. Note however that the price to pay with the proposed marginalized particle filter is a higher computational cost, essentially when compared to other EKF based methods. The proposed MPF algorithm can serve as a reference for ECG denoising which is interesting in many biomedical applications. Future work includes applying a similar sequential Monte Carlo method to a different Bayesian model recently introduced in [11] for ECG delineation.

REFERENCES

[1] A. Gruetzmann, S. Hansen and J. Müller, "Novel dry electrodes for ECG monitoring," *Physiol. Meas.*, vol. 28, pp. 1375-1390, 2007.

[2] P. E. McSharry, G. D. Clifford, L. Tarassenko, and L. A. Smith, "A Dynamic Model for Generating Synthetic Electrocardiogram Signals," *IEEE Trans. Biomed. Eng.*, vol. 50, no. 3, pp. 289-294, Mar. 2003.

[3] R. Sameni, M. B. Shamsollahi, C. Jutten, and G. D. Clifford, "A nonlinear Bayesian filtering framework for ECG denoising," *IEEE Trans. Biomed. Eng.*, vol. 54, no. 12, pp. 2172-2185, Dec. 2007.

[4] O. Sayadi, R. Sameni, and M. B. Shamsollahi, "ECG Denoising Using Parameters of ECG Dynamical Model as the States of an Extended Kalman Filter," in *Proc. 29th Annu. Int. Conf. IEEE Eng. Medicine Biol. Soc. (EMBC)*, Lyon, France, Aug. 23-26, 2007, pp. 2548-2551.

[5] O. Sayadi and M. B. Shamsollahi, "ECG Denoising and Compression Using a Modified Extended Kalman Filter Structure," *IEEE Trans. Biomed. Eng.*, vol. 55, no. 9, pp. 2240-2248, Sep. 2008.

[6] M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking," *IEEE Trans. Signal Process.*, vol. 50, no. 2, pp. 174-188, Feb. 2002.

[7] A. Doucet, N. de Freitas and N. Gordon, *Sequential Monte Carlo Methods in Practice*, Springer-Verlag, 2001.

[8] A. Doucet, N. Gordon and V. Krishnamurthy, "Particle filters for state estimation of jump Markov linear systems," *IEEE Trans. Signal Process.*, vol. 49, no. 3, pp. 613-624, 2001.

[9] T. Schön, F. Gustafsson, and P.-J. Nordlund, "Marginalized Particle Filters for Mixed Linear/Nonlinear State-Space Models," *IEEE Trans. Signal Process.*, vol. 53, no. 7, pp. 2279-2289, Jul. 2005.

[10] *The MIT-BIH Normal Sinus Rhythm Database.* Available online at http://www.physionet.org/physiobank/database/nsrdb/.

[11] C. Lin, C. Mailhes and J.-Y. Tourneret "P- and T-wave delineation in ECG signals using a Bayesian approach and a partially collapsed Gibbs sampler," *IEEE Trans. Biomed. Eng.*, vol. 57, no. 12, pp. 2840-2849, Dec. 2010.