T-wave Alternans Detection Using a Bayesian Approach and a Gibbs Sampler

Chao Lin, Corinne Mailhes and Jean-Yves Tourneret

***Abstract*— The problem of detecting T-wave alternans (TWA) in ECG signals has received considerable attention in the biomedical community. This paper introduces a Bayesian model for the T waves contained in ECG signals. A block Gibbs sampler was recently studied to estimate the parameters of this Bayesian model (including wave locations, amplitudes and shapes). This paper shows that the samples generated by this Gibbs sampler can be used efficiently for TWA detection via different statistical tests constructed from odd and even T-wave amplitude samples. The proposed algorithm is evaluated on real ECG signals subjected to synthetic TWA and compared with two classical algorithms.**

***Index Terms*— T-wave alternans, Bayesian analysis, Gibbs sampler.**

I. INTRODUCTION

T-wave alternans (TWA) is a phenomenon appearing in electrocardiograms (ECGs) as an index of malignant arrhythmias and sudden cardiac death [1]. TWA is defined as a consistent fluctuation in the repolarization morphology which repeats on an every-other-beat basis. Since the first report of non-visible (microvolt-level) TWA by Adam $et\ al.$ in the 1980s, intensive research has been conducted on developing TWA detection and estimation algorithms. A complete and comprehensive review of signal processing methods to detect and estimate TWA proposed before 2005 can be found in [2]. More recently alternative techniques include the multilead TWA detection by using principal component analysis [3] and an empirical-mode decomposition based method [4]. The fact that TWA amplitude is in the range of microvolts, together with the presence of the baseline and the physiological noise in the ECG make the TWA detection a difficult task. The main drawback of existing TWA analysis approaches is either their sensitivity to the presence of nonalternant components with high amplitude or their poor sensitivity to low-level TWA. Another problem with existing methods is that they generally require preprocessing steps for baseline suppression, rough segmentation or alignment of ST-T complexes. Thus, their performance is strongly influenced by the quality of these preprocessing procedures.

A Bayesian model was recently introduced in [5] for P- and T-wave delineation. This model takes into account prior distributions for the unknown parameters (wave locations and amplitudes as well as waveform and local baseline coefficients). These prior distributions are combined with the likelihood of the observed data to define the posterior distribution of the unknown parameters. Simulation methods have then shown interesting properties to alleviate the complexity of this posterior distribution. In particular, a Gibbs sampler was proposed in [5] to generate samples distributed according to the posterior and to estimate the model parameters using these samples. The present paper introduces new TWA detection methods based on the data generated by this Gibbs sampler. More precisely, the previous proposed algorithm [5] is slightly modified in order to perform T-wave delineation taking into account a distinction between odd and even beats. Then odd and even T-wave amplitudes generated by the sampler can be used to build statistical tests for TWA detection. This paper concentrates on two tests: the two-sample Kolmogorov-Smirnov test (which is a non-parametric and robust method for comparing two samples) and the two-sample Student's t-test which is based on the assumption of normality for comparing the mean of two samples [6]. Note that contrary to the statistical test proposed in [7], the proposed method computes multiple test statistics for each observation window (one per iteration of the Gibbs sampler) that can be used advantageously to derive detection performance (detection probability, probability of false alarm, receiver operational characteristics, etc.). Compared with a test based on a single estimation, the proposed method provides information about the reliability of the detection which is important for medical diagnostics.

The paper is organized as follows. Section II summarizes the model used for the T waves contained in the ECG signal. Section III summarizes the prior and posterior distributions of the unknown model parameters studied in [5]. TWA detection strategies based on the estimation of the odd and even T-wave amplitudes are then presented in Section IV. Simulation results performed on simulated TWA affecting real ECG signals and a comparison with two standard TWA detection algorithms are presented in Section V.

II. SIGNAL MODEL

Most TWA detection methods are based on consecutive extracted and aligned T waves. Each T wave is usually selected with a fixed or RR-adjusted time window and aligned by using an appropriate technique. However, as explained in [8], TWA analysis is affected by the performance of the alignment techniques since T-wave delineator used for TWA analysis must show inter-beat stability in the fiducial point determination. In this paper, we propose a new signal model for the extracted T waves that allow alignment errors to be compensated. In the proposed method, we first detect QRS-complexes that are the most prominent parts of the ECG signal, and we shift a nonoverlapping $2D$-beat processing window to cover the whole signal. In the processing window, the right hand neighborhood of each successive pair of QRS-offset constitutes a T-wave search interval (shown in Fig. 1(a)). The length of the nth T-wave search interval $N_{\mathrm{T},n}$ can be fixed either according to the cardiologists or simply as a fixed percentage of N_n, which is the length of a non-QRS interval ($n \in \{1, \ldots, 2D\}$). The T-wave search intervals are divided into odd T-wave blocks denoted as $\mathcal{J}_{\mathrm{o}} = \{\mathcal{J}_{\mathrm{o},1}, \cdots \mathcal{J}_{\mathrm{o},D}\}$ (containing the D odd T-wave search intervals) and even T-wave blocks denoted as $\mathcal{J}_{\mathrm{e}} = \{\mathcal{J}_{\mathrm{e},1}, \cdots \mathcal{J}_{\mathrm{e},D}\}$ (containing the even intervals).

As shown in Fig. 1(b), signals within each T-wave search interval can be approximated by one main pulse representing the T wave plus a local baseline. The odd (resp. even) waveforms are assumed to be constant within each processing window, contrary to the amplitudes and locations which vary with n. Therefore, T waves within D odd search intervals can be modeled by the convolution of an unknown waveform $\boldsymbol{h}_{\mathrm{o}} = (h_{\mathrm{o},0} \cdots h_{\mathrm{o},L})^T$ (of length L) with an unknown "impulse" sequence $\boldsymbol{u}_{\mathrm{o}} = (u_{\mathrm{o},1} \cdots u_{\mathrm{o},M})^T$ indicating

C. Lin is with the TéSA Laboratory, University of Toulouse, Toulouse 31071, France, chao.lin@tesa.prd.fr

C. Mailhes and J.-Y. Tourneret are with the Institut de recherche en informatique de Toulouse (UMR 5505 of the CNRS), University of Toulouse, Toulouse 31071, France, corinne.maihles@enseeiht.fr; jean-yves.tourneret@enseeiht.fr

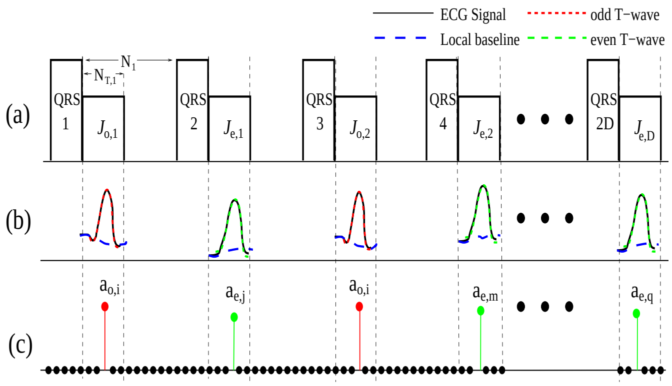

Fig. 1. (a) T-wave search intervals within the $2D$-beat processing window. (b) T waves within each non-QRS region. (c) T-wave amplitudes. Here we set $N_{\mathrm{T},n} = N_n/2$.

the odd T-wave locations and amplitudes (see Fig. 1(c)). The impulse sequences can be defined as the products $u_{\mathrm{o},k} = b_{\mathrm{o},k} a_{\mathrm{o},k}$ of binary indicator sequences $b_{\mathrm{o},k} \in \{0, 1\}$ and amplitude factors $a_{\mathrm{o},k} \in \mathbb{R}$. Each $b_{\mathrm{o},k} = 1$ indicates the location of an odd T wave, and the corresponding $a_{\mathrm{o},k}$ is the respective amplitude. Note that the $a_{\mathrm{o},k}$ are undefined for all k where $b_{\mathrm{o},k} = 0$. Similarly, the even T waves within a window are modeled by the convolution of $\boldsymbol{h}_{\mathrm{e}} = (h_{\mathrm{e},0} \cdots h_{\mathrm{e},L})^T$ with $\boldsymbol{u}_{\mathrm{e}} = (u_{\mathrm{e},1} \cdots u_{\mathrm{e},M})^T$. Let K denote the corresponding signal length. The ECG signal within the processing window can then be written as

$$x_k = \sum_{l=0}^{L} h_{\mathrm{o},l} \left(a_{\mathrm{o},k-l} b_{\mathrm{o},k-l}\right) + \sum_{l=0}^{L} h_{\mathrm{e},l} \left(a_{\mathrm{e},k-l} b_{\mathrm{e},k-l}\right) + c_k + w_k \quad (1)$$

where c_k defines the baseline and w_k is the additive white Gaussian noise with unknown variance σ_w^2. Note that only indexes k belonging to a T-wave search interval are considered in this paper.

Baseline removal is generally recognized as an important processing step which makes the estimation of local baseline essential for TWA detection. Here, we propose to model the local baseline within the nth interval $\mathcal{J}_n$ by using a 4th-degree polynomial, i.e.,

$$c_{n,k} = \sum_{i=1}^{5} \gamma_{n,i} k^{i-1}, k = 1, \ldots, N_n \,, \quad (2)$$

for each $n \in \{1, \ldots, 2D\}$. In vector-matrix form, (2) can be written as $\boldsymbol{c}_n = \boldsymbol{M}_n \boldsymbol{\gamma}_n$, where $\boldsymbol{M}_n$ is a known $N_n \times 5$ Vandermonde matrix and $\boldsymbol{\gamma}_n = (\gamma_{n,1} \cdots \gamma_{n,5})^T$ contains the unknown baseline coefficients. The baseline sequence for the entire $2D$-beat window can then be written as $\boldsymbol{c} = (\boldsymbol{c}_1, \ldots, \boldsymbol{c}_{2D}) = \boldsymbol{M}\boldsymbol{\gamma}$ where $\boldsymbol{M}$ is a $K \times 10D$ matrix and $\boldsymbol{\gamma}$ is a $10D \times 1$ vector.

Let $\boldsymbol{b}_{\mathrm{o}}$, $\boldsymbol{b}_{\mathrm{e}}$, $\boldsymbol{a}_{\mathrm{o}}$, and $\boldsymbol{a}_{\mathrm{e}}$ denote the $M \times 1$ vectors corresponding to $b_{\mathrm{o},k}$, $b_{\mathrm{e},k}$, $a_{\mathrm{o},k}$, and $a_{\mathrm{e},k}$, and $\boldsymbol{B}_{\mathrm{o}} \triangleq \mathrm{diag}(\boldsymbol{b}_{\mathrm{o}})$, $\boldsymbol{B}_{\mathrm{e}} \triangleq \mathrm{diag}(\boldsymbol{b}_{\mathrm{e}})$ denote the diagonal $M \times M$ matrices whose diagonal elements are formed by the components of $\boldsymbol{b}_{\mathrm{o}}$ and $\boldsymbol{b}_{\mathrm{e}}$. By concatenating (1) for $k = 1, \ldots, K$, where K is the number of ECG signal samples, the following matrix equation can be obtained

$$\boldsymbol{x} = \boldsymbol{F}_{\mathrm{o}} \boldsymbol{B}_{\mathrm{o}} \boldsymbol{a}_{\mathrm{o}} + \boldsymbol{F}_{\mathrm{e}} \boldsymbol{B}_{\mathrm{e}} \boldsymbol{a}_{\mathrm{e}} + \boldsymbol{M}\boldsymbol{\gamma} + \boldsymbol{w} \,, \quad (3)$$

where $\boldsymbol{F}_{\mathrm{o}}$ is the Toeplitz matrix of size $K \times M$ with first row $[h_{o,0} \; \mathbf{0}_{M-1}]$ and first column $\left[(\boldsymbol{h}_{\mathrm{o}})^T \; \mathbf{0}_{M-1}\right]^T$, while $\boldsymbol{F}_{\mathrm{e}}$ is the Toeplitz matrix of size $K \times M$ with first row $[h_{e,0} \; \mathbf{0}_{M-1}]$ and first column $\left[(\boldsymbol{h}_{\mathrm{e}})^T \; \mathbf{0}_{M-1}\right]^T$ ($\mathbf{0}_{M-1}$ is the $(M-1) \times 1$ vector of zeros).

III. BAYESIAN MODEL

The unknown parameter vector resulting from the above parametrization is $\boldsymbol{\theta} = (\boldsymbol{\theta}_{\mathrm{o}}^T \; \boldsymbol{\theta}_{\mathrm{e}}^T \; \boldsymbol{\theta}_{\mathrm{cw}}^T)^T$, where $\boldsymbol{\theta}_{\mathrm{o}} \triangleq (\boldsymbol{b}_{\mathrm{o}}^T \; \boldsymbol{a}_{\mathrm{o}}^T \; \boldsymbol{h}_{\mathrm{o}}^T)^T$ and $\boldsymbol{\theta}_{\mathrm{e}} \triangleq (\boldsymbol{b}_{\mathrm{e}}^T \; \boldsymbol{a}_{\mathrm{e}}^T \; \boldsymbol{h}_{\mathrm{e}}^T)^T$ are related to the odd and even T waves, and $\boldsymbol{\theta}_{\mathrm{cw}} \triangleq (\boldsymbol{\gamma}^T \; \sigma_w^2)^T$ is related to the baseline and noise. Bayesian detection/estimation relies on the posterior distribution $p(\boldsymbol{\theta}|\boldsymbol{x}) \propto p(\boldsymbol{x}|\boldsymbol{\theta})p(\boldsymbol{\theta})$ (here, $\propto$ means "proportional to"), where $p(\boldsymbol{x}|\boldsymbol{\theta})$ is the likelihood function and $p(\boldsymbol{\theta})$ is the prior distribution of $\boldsymbol{\theta}$.

Likelihood function. Using our model (3) and the fact that $\boldsymbol{\omega}$ is white Gaussian, the likelihood function is obtained as

$$p(\boldsymbol{x}|\boldsymbol{\theta}) \propto \frac{1}{\sigma_w^K} \exp\!\left(-\frac{1}{2\sigma_w^2} \|\boldsymbol{x} - \boldsymbol{F}_{\mathrm{o}}\boldsymbol{B}_{\mathrm{o}}\boldsymbol{a}_{\mathrm{o}} - \boldsymbol{F}_{\mathrm{e}}\boldsymbol{B}_{\mathrm{e}}\boldsymbol{a}_{\mathrm{e}} - \boldsymbol{M}\boldsymbol{\gamma}\|^2\right),$$

where $\|\boldsymbol{x}\|^2 = \boldsymbol{x}^T\boldsymbol{x}$.

Prior distributions. Since there are no known relations between $(\boldsymbol{b}_{\mathrm{o}}, \boldsymbol{a}_{\mathrm{o}})$, $(\boldsymbol{b}_{\mathrm{e}}, \boldsymbol{a}_{\mathrm{e}})$, $\boldsymbol{\alpha}_{\mathrm{o}}$, $\boldsymbol{\alpha}_{\mathrm{e}}$, $\boldsymbol{\gamma}$, and σ_n^2, all these sets of parameters are assumed to be *a priori* statistically independent. We will now discuss the prior distributions of these parameters. Let $\boldsymbol{b}_{\mathcal{J}_{\mathrm{o},n}}$, $n \in \{1, \ldots, D\}$ contain all entries of the odd T-wave indicator vector $\boldsymbol{b}_{\mathrm{o}}$ that are indexed by the odd T-wave interval $\mathcal{J}_{\mathrm{o},n}$. The indicators are subject to a *block constraint*: within $\mathcal{J}_{\mathrm{o},n}$, there is one T wave (i.e., $\|\boldsymbol{b}_{\mathcal{J}_{\mathrm{o},n}}\| = 1$) or none (i.e., $\|\boldsymbol{b}_{\mathcal{J}_{\mathrm{o},n}}\| = 0$), the latter case being very unlikely. Therefore, we define the prior of $\boldsymbol{b}_{\mathcal{J}_{\mathrm{o},n}}$ as

$$p(\boldsymbol{b}_{\mathcal{J}_{\mathrm{o},n}}) = \begin{cases} p_0 & \text{if } \|\boldsymbol{b}_{\mathcal{J}_{\mathrm{o},n}}\| = 0 \\ p_1 & \text{if } \|\boldsymbol{b}_{\mathcal{J}_{\mathrm{o},n}}\| = 1 \\ 0 & \text{otherwise,} \end{cases} \quad (4)$$

where $p_1 = (1 - p_0)/N_{\mathrm{o},n}$ and p_0 is chosen very small. The indicators $\boldsymbol{b}_{\mathcal{J}_{\mathrm{o},n}}$ are supposed independent, and all remaining entries of the total vector $\boldsymbol{b}_{\mathrm{o}}$ (i.e., entries outside the search intervals $\mathcal{J}_{\mathrm{o}}$) are zero. Thus, the prior of $\boldsymbol{b}_{\mathrm{o}}$ is the product of the priors $p(\boldsymbol{b}_{\mathcal{J}_{\mathrm{o},n}})$.

For the T-wave amplitudes $a_{\mathrm{o},k}$ corresponding to $b_{\mathrm{o},k} = 1$ (recall that the $a_{\mathrm{o},k}$ are undefined otherwise), we choose a zero-mean Gaussian prior, i.e., $p(a_{\mathrm{o},k}|b_{\mathrm{o},k} = 1) = \mathcal{N}(0, \sigma_a^2)$. This allows for both positive and negative amplitudes. Amplitudes at different k are modeled as statistically independent. It follows that $u_{\mathrm{o},k} = b_{\mathrm{o},k} a_{\mathrm{o},k}$ is the kth element of a Bernoulli-Gaussian sequence with block constraints. The priors of the even T-wave indicators $b_{\mathrm{e},k}$ and amplitudes $a_{\mathrm{e},k}$ are defined in a fully analogous way, with the same fixed hyperparameters p_0, p_1, and σ_a^2. Moreover, the even T-wave variables are supposed to be independent of the odd T-wave variables. The odd T-waveform vector is assigned a zero-mean Gaussian prior, i.e., $p(\boldsymbol{h}_{\mathrm{o}}) = \mathcal{N}(\mathbf{0}, \sigma_{\boldsymbol{h}}^2 \mathbf{I}_{L+1})$, where $\mathbf{I}_{L+1}$ denotes the identity matrix of size $(L+1) \times (L+1)$. The same prior is chosen for the even T-wave coefficients, i.e., $p(\boldsymbol{h}_{\mathrm{e}}) = \mathcal{N}(\mathbf{0}, \sigma_{\boldsymbol{h}}^2 \mathbf{I}_{L+1})$. The baseline coefficients $\gamma_{n,i}$ are also modeled as independent identically distributed zero-mean Gaussian, i.e., $p(\boldsymbol{\gamma}) = \mathcal{N}(\mathbf{0}, \sigma_\gamma^2 \mathbf{I}_{5D})$. The reader is invited to consult [5] for details about the hyperparameter priors.

Posterior distribution. The posterior of the parameter vector $\boldsymbol{\theta}$ is given by

$$p(\boldsymbol{\theta}|\boldsymbol{x}) \propto p(\boldsymbol{x}|\boldsymbol{\theta})p(\boldsymbol{\theta}) = p(\boldsymbol{x}|\boldsymbol{\theta})p(\boldsymbol{\theta}_{\mathrm{o}})p(\boldsymbol{\theta}_{\mathrm{e}})p(\boldsymbol{\theta}_{\mathrm{cw}}) \,, \quad (5)$$

with $p(\boldsymbol{\theta}_{\mathrm{cw}}) = p(\boldsymbol{\gamma})p(\sigma_w^2)$, $p(\boldsymbol{\theta}_{\mathrm{o}}) = p(\boldsymbol{a}_{\mathrm{o}}|\boldsymbol{b}_{\mathrm{o}})p(\boldsymbol{b}_{\mathrm{o}})p(\boldsymbol{h}_{\mathrm{o}})$ and $p(\boldsymbol{\theta}_{\mathrm{e}}) = p(\boldsymbol{a}_{\mathrm{e}}|\boldsymbol{b}_{\mathrm{e}})p(\boldsymbol{b}_{\mathrm{e}})p(\boldsymbol{h}_{\mathrm{e}})$. Due to the complexity of this distribution, a block Gibbs sampler generating samples asymptotically distributed according to $p(\boldsymbol{\theta}|\boldsymbol{x})$ was studied in [5]. From these samples, the discrete parameters $\boldsymbol{b}_{\mathrm{o}}$ and $\boldsymbol{b}_{\mathrm{e}}$ can be detected by means of the sample-based maximum a posteriori (MAP) detector whereas the continuous parameters $\boldsymbol{a}_{\mathrm{o}}$, $\boldsymbol{a}_{\mathrm{e}}$, $\boldsymbol{h}_{\mathrm{o}}$, $\boldsymbol{h}_{\mathrm{e}}$, $\boldsymbol{\gamma}$, and σ_w^2 can be estimated by means of the sample-based minimum mean square error estimator (see [5] for details).

IV. BAYESIAN TWA DETECTION

As explained in Section III, the T-wave locations and amplitudes can be sampled according to their joint posterior within each T-wave search interval. Taking advantage of the Gibbs sampling method, different statistical tests can be carried out on the T-wave amplitudes generated by the proposed sampler to detect TWA[1]. First, we consider the two-sample Kolmogorov-Smirnov (KS) test which is a classical nonparametric method for comparing two samples. Let $\boldsymbol{a}_{\mathrm{o}}^{(i)} = \left(a_{\mathrm{o},1}^{(i)}, \ldots, a_{\mathrm{o,D}}^{(i)}\right)^T$ and $\boldsymbol{a}_{\mathrm{e}}^{(i)} = \left(a_{\mathrm{e},1}^{(i)}, \ldots, a_{\mathrm{e,D}}^{(i)}\right)^T$ denote the odd and even T-wave amplitudes within the $2D$-beat window generated at the i-th iteration of the Gibbs sampler. The TWA detection can be formulated as the following binary hypothesis test

$$\mathcal{H}_0 : F_{\mathrm{o}} = F_{\mathrm{e}}, \quad \mathcal{H}_1 : F_{\mathrm{o}} \neq F_{\mathrm{e}}$$

where F_{o} and F_{e} are the cumulative distribution functions of the odd and even T-wave amplitude samples. The KS test statistic is defined as

$$s^{(i)} = \sup_x \left| \hat{F}_{\mathrm{o}}^{(i)}(x) - \hat{F}_{\mathrm{e}}^{(i)}(x) \right| \tag{6}$$

where $\hat{F}_{\mathrm{o}}^{(i)}$ and $\hat{F}_{\mathrm{e}}^{(i)}$ are the empirical distribution functions of $\boldsymbol{a}_{\mathrm{o}}^{(i)}$ and $\boldsymbol{a}_{\mathrm{e}}^{(i)}$, respectively.

The two-sample Student's t-test can also be applied to compare the means of the two samples $\boldsymbol{a}_{\mathrm{o}}^{(i)}$ and $\boldsymbol{a}_{\mathrm{e}}^{(i)}$. The TWA detection can be then formulated as

$$\mathcal{H}_0 : \mu_{\mathrm{o}} = \mu_{\mathrm{e}}, \quad \mathcal{H}_1 : \mu_{\mathrm{o}} \neq \mu_{\mathrm{e}}$$

where μ_{o} and μ_{e} are the means of the odd and even T-wave amplitude samples. The t-test statistic can be computed as follows

$$t^{(i)} = \frac{\overline{\boldsymbol{a}}_{\mathrm{o}}^{(i)} - \overline{\boldsymbol{a}}_{\mathrm{e}}^{(i)}}{S_{\mathrm{eo}}^{(i)} \sqrt{\frac{2}{D}}} \tag{7}$$

where $\overline{\boldsymbol{a}}_{\mathrm{o}}^{(i)} = \frac{1}{D}\sum_{j=1}^{D} a_{\mathrm{o,j}}^{(i)}$, $\overline{\boldsymbol{a}}_{\mathrm{e}}^{(i)} = \frac{1}{D}\sum_{j=1}^{D} a_{\mathrm{e,j}}^{(i)}$ and

$$S_{\mathrm{eo}}^{(i)} = \sqrt{\frac{1}{2D-2}\left(\sum_{j=1}^{D}\left(a_{\mathrm{o,j}}^{(i)} - \overline{\boldsymbol{a}}_{\mathrm{o}}^{(i)}\right)^2 + \sum_{j=1}^{D}\left(a_{\mathrm{e,j}}^{(i)} - \overline{\boldsymbol{a}}_{\mathrm{e}}^{(i)}\right)^2\right)}$$

By computing the test statistics (6) or (7) at each iteration of the Gibbs sampler, we obtain $N_{\mathrm{eff}} = N_{\mathrm{r}} - N_{\mathrm{bi}}$ (N_{r} is the number of iterations after convergence and N_{bi} is the number of burn-in iterations) samples of the test statistics corresponding to the same $2D$-beat block and we can thus approximate the distribution of the test statistic using these samples. The final test decision can be made based on the percentage of the obtained statistics that reject the null hypothesis (absence of TWA). An interesting property of the proposed tests is that their reliability can be evaluated, e.g., by computing the detection probability τ for any processing window

$$\tau = \frac{\text{Number of test statistic samples rejecting hypothesis } \mathcal{H}_0}{\text{Total number of test statistic samples}}$$

This reliability information about the decision can be useful for medical diagnostics.

[1]Note that the first iterations belonging to the so-called burn-in period are not considered for parameter estimation or for TWA detection.

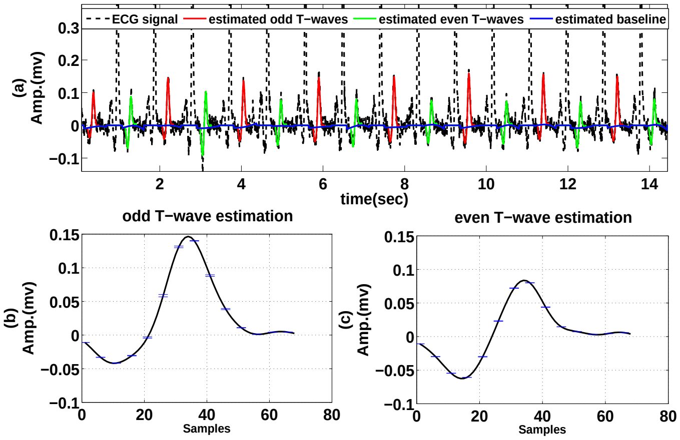

Fig. 2. (a) Segment of dataset "e0303" with synthetic TWA and "ma" noise SNR=10 dB (black), estimated local baseline (blue), and estimated odd (red) and even (green) T waves. (b) Odd T-wave estimation averages (black) and the corresponding confidence intervals (blue) for the 16-beat window. (c) Even T-wave estimation averages (black) and the corresponding confidence intervals (blue) for the 16-beat window.

V. SIMULATION RESULTS

Biomedical signal processing techniques are usually evaluated on standard databases, where the output of the technique is compared to manual expert annotations. However, because TWA is often non visible due to its low amplitude (sometimes below the noise level), the lack of validation databases has been a major problem for TWA analysis. Simulated alternans with real nonalternant ECG recordings are widely used in the community [2]. In the following simulations, 20 healthy ECG segments (with 128 beats) have been selected from different databases. TWA episodes are simulated by adding and subtracting alternatively (on a every-other-beat basis) a Hanning window to the delineated T waves as in [4]. A small TWA amplitude value of V_{alt}=35μV has been chosen for the evaluation. Two different physiological noise sources have been considered to evaluate the two proposed TWA detectors under real noise conditions: electrode motion ("em") and muscular activity ("ma"). Note that the "em" and "ma" noises have been extracted from the MIT-BIH noise stress test database. As a preprocessing step, the QRS complexes have been detected using the algorithm proposed in [9]. Based on the detected QRS complex locations, T-wave search intervals have been defined. The processing window length has been set to $2D$=16 beats, which is the smallest window length among the methods mentioned in [2]. Note that having a good detection performance with a small window length is beneficial for medical diagnostics. The Gibbs sampler studied in [5] has been run for each processing window with $N_{\mathrm{bi}} = 40$ burn-in iterations and $N_{\mathrm{eff}} = 100$ iterations to compute the estimates.

Fig. 2 shows the estimation results for an ECG signal segment from the European ST-T dataset "e0303" with synthetic TWA. The "ma" noise has been added to the signal with SNR=10 dB. Typical estimates for the baseline and odd/even T waves are depicted in Fig. 2(a). Fig. 2(b) and (c) show the averages of T-wave estimates resulting from 20 Monte Carlo runs and the corresponding confidence intervals (error bars) for the odd and even beats within the 16-beats window. As explained in Section IV, the KS test and t-test statistics can be determined for each iteration of the Gibbs sampler according to (6) and (7) providing $N_{\mathrm{eff}} = 100$ decisions for each processing window. Fig. 3 shows representative situations for three different processing windows. Fig. 3 (a) and (d) show the

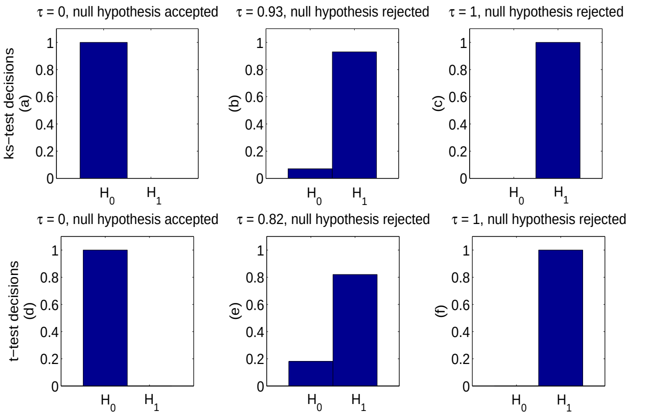

Fig. 3. The KS-test (top) and the t-test (bottom) decisions made for three different 16-beat windows: (a) and (d) show the test decisions for one window of dataset "e0303" with no synthetic TWA and "ma" noise SNR=10dB; (b) and (e) show the test decisions for one window of dataset "e0303" with synthetic 35 μv TWA and "ma" noise SNR=5dB; (c) and (f) show the test decisions for one window of dataset "e0303" with synthetic 35 μv TWA and "ma" noise SNR=10dB.

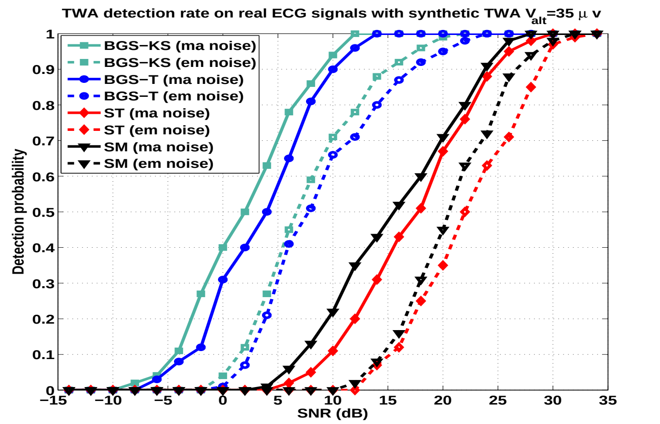

Fig. 4. Detection performance for real ECGs with synthetic $35\mu V$ TWA. The proposed Bayesian Gibbs sampler with KS test (green square markers), Bayesian Gibbs sampler with t-test (blue round markers), the ST method (red diamond markers) and the SM (black triangle markers) are tested in both "ma" (continuous lines) and "em" (dotted lines) noise conditions.

KS test and t-test decisions for one window of dataset "e0303" with no synthetic TWA and "ma" noise (SNR=10dB). As can be seen, both tests have accepted the null hypothesis 100 times, therefore the null hypothesis can be accepted with full certainty ($\tau = 0$) for this window. Fig. 3 (c) and (f) show the decisions for one window of dataset "e0303" corrupted with synthetic 35 μv TWA and "ma" noise (SNR=10dB). The null hypothesis can be rejected with full certainty ($\tau = 1$) since both tests have rejected the null hypothesis 100 times. Fig. 3 (b) and (e) show results for a more complicated case where one window of dataset "e0303" is corrupted with synthetic 35 μv TWA and with a higher "ma" noise level than in the previous case (SNR=5dB). For this window, the null hypothesis can be rejected with detection probabilities of $\tau = 0.93$ and $\tau = 0.82$ for the KS test and the t-test, respectively. It can be seen that, depending on signal characteristics (presence of physiological noise, baseline behavior ...), the null hypothesis rejection rate can be exactly unity (Fig. 3 (c) and (f)) or just below 1 (Fig. 3 (b) and (e)). Note again that using several samples from the Gibbs sampling iterations provides multiple test statistics allowing decision with an interesting reliability information.

For a quantitative comparison, we have implemented two classical methods, the spectral method (SM) [10] and the statistical test based on the maximum amplitude of the ST-T complex (ST) [7]. Fig. 4 shows the detection results achieved with real signals with synthetic $35\mu V$ TWA corrupted by "ma" and "em" noises for the Bayesian Gibbs sampler with KS test (BGS-KS) and t-test (BGS-T), the SM and the ST method. Note that 20 Monte Carlo runs have been carried out for each SNR value, where the noise realizations have been changed from one simulation to another. The processing window length has been set to 16 beats for all the methods. As can be seen, Bayesian Gibbs sampler based tests yield better results for both "ma" and "em" noise compared to ST and SM (e.g., an improvement of 10dB is achieved for having $P_{\text{D}} = 1$). The KS test gives slightly better results than the t-test. Note however that the proposed methods have higher computational costs especially when using large processing windows (e.g., 128 beats). Thus their use is generally recommended for small processing windows.

VI. Conclusions

This paper studied a TWA detection technique based on a Gibbs sampler recently introduced in [5] for generating samples distributed according to the posterior of an appropriate Bayesian model. Benefiting from the Gibbs sampling, we carried out two different statistical tests for comparing the odd and even groups of T-wave amplitude samples. Validation on real ECG signals with synthetic TWA showed that the proposed method provides reliable TWA detection and accurate TWA waveform estimation for a wide variety of wave morphologies with different types of noise. Moreover, the proposed method provided better result than two reference methods, at the price of higher computational complexity. An interesting property of the proposed strategy is that additional information concerning the reliability of the detection can be obtained. Current investigations include the characterization of TWA waveforms (i.e., by computing the difference between the odd and even T-wave estimations presented in Fig. 2(b) and (c)), which is important to detect arrhythmic risk. Depending on the TWA characteristics, other statistical tests could also be studied.

References

[1] D. S. Rosenbaum, L. E. Jackson, J. M. Smith, H. Garan, J. N. Ruskin, and R. J. Cohen, "Electrical alternans and vulnerability to ventricular arrhythmias," *N. Engl. J. Med.*, vol. 330, no. 4, pp. 235–241, 1994.

[2] J. P. Martínez and S. Olmos, "Methodological principles of T Wave Alternans analysis: A unified framework," *IEEE Trans. Biomed. Eng.*, vol. 52, no. 4, pp. 599–613, 2004.

[3] V. Monasterio, P. Laguna, and J. P. Martínez, "Multilead analysis of T-wave alternans in the ECG using principal component analysis," *IEEE Trans. Biomed. Eng.*, vol. 56, no. 7, pp. 1880–1890, 2009.

[4] M. Blanco-Velasco, F. Cruz-Roldán, J. I. Godino-Llorente, and K. E. Barner, "Nonlinear trend estimation of the ventricular repolarization segment for T-wave alternans detection," *IEEE Trans. Biomed. Eng.*, vol. 57, no. 10, pp. 2402–2412, 2010.

[5] C. Lin, C. Mailhes, and J.-Y. Tourneret, "P- and T-wave delineation in ECG signals using a Bayesian approach and a partially collapsed Gibbs sampler," *IEEE Trans. Biomed. Eng.*, vol. 57, no. 12, pp. 2840–2849, 2010.

[6] E. L. Lehmann, *Testing Statistical Hypotheses*. New York: Springer, 1997.

[7] T. Srikanth, D. Lin, N. Kanaan, and H. Gu, "Presence of T wave alternans in the statistical context-a new approach to low amplitude alternans measurement," *Comput. Cardiol.*, vol. 29, pp. 681–684, 2002.

[8] S. M. Narayan and J. M. Smith, "Spectral analysis of periodic fluctuations in electrocardiographic repolarization," *IEEE Trans. Biomed. Eng.*, vol. 46, pp. 203–212, 1999.

[9] J. Pan and W. J. Tompkins, "A real-time QRS detection algorithm," *IEEE Trans. Biomed. Eng.*, vol. 32, no. 3, pp. 230–236, 1985.

[10] J. M. Smith, E. A. Clancy, C. R. Valeri, J. N. Ruskin, and R. J. Cohen, "Electrical alternans and cardiac electrical instability," *Circulation*, vol. 77, no. 1, pp. 110–121, 1994.